Mind Map–2

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ELECTROSTATIC POTENTIAL AND CAPACITANCE

Electric potential due to a charged conducting spherical shell

At a point on the surface or inside the spherical shell

$V = \frac{1}{4\pi \epsilon_0}\frac{q}{R}(r \le R)$

At a point outside the spherical shell

$V = \frac{1}{4\pi \epsilon_0}\frac{q}{r}(r > R)$

Electric potential due to a charged non-conducting sphere

At a point on the surface or inside the sphere

$= \frac{1}{4\pi \epsilon_0}\frac{q}{R}(r \le R)$

At a point outside the non-conducting sphere

$V = \frac{1}{4\pi \epsilon_0}\frac{q}{r}(r > R)$

Electrostatic potential due to infinite thin plan sheet or charge

$V = -\frac{\sigma r}{2\varepsilon_0} + C$

σ = uniform surface charge density

Electric potential due to a dipole

On axial line

$V = \frac{1}{4\pi \epsilon_0}\frac{p}{r^2}$

On equatorial line V = 0

At general point,

$V_g = \frac{kP\cos\theta}{r^2}$

Electrostatic potential

$(V_0) = \frac{\text{work done } w_\infty}{\text{charge } (q_0)}$

Positive potential due to +(ve) charge and negative potential due to –(ve) charge

Due to charged circular ring

(i) At a point distance x away from the centre of the ring

$V = \frac{kQ}{\sqrt{x^2 + k^2}}$

(ii) At centre,

$V_{\text{centre}} = \frac{kQ}{k}$

Electrostatic potential energy of a

(i) System of 'n' charges $U = -\frac{K}{2}\sum_{\substack{i,j \\ i \ne f}}^{n} \frac{Q_1 Q_2}{r_{if}}$

(ii) Uniformly charged sphere, $U = \frac{30Q^2}{20\pi \epsilon_0 R}$

(iii) Uniformly charged thin spherical shell $U = \frac{Q^2}{8\pi \epsilon_0 R}$

(iv) Energy density $U_e = \frac{1}{2}\varepsilon_0 \epsilon^2$

Equipotential surface

Imaginary surface joining the points of same potential in an electric field

- The direction of electric field is perpendicular to the equipotential surface or lines.
- A metallic surface of any shape is an equipotential surface.

Electrostatic potential due to a point charge

$V = K\frac{q}{r}$

Electrostatic potential due to a system of charges

$V = V_1 + V_2 + V_3 \ldots + V_n$

$V = K\sum_{i=1}^{n}\frac{q_i}{r_i}$

Electrostatic potential due to continuous charge distribution

$V - \int dV - \int \frac{dQ}{4\pi \epsilon_0 r}$

Relation between electric potential and filed,

$E = -\frac{dV}{dr}$

Negative of the slope of the v–r graph denotes intensity of electric field, tanθ

$\tan\theta = \frac{V}{r} = -E$

Combination of Charged Drops

If n identical drops each having radius r Capacitance, c, Charge, q, Potential, v and energy, u. If these drops are combined to form a big drop of radius, R,

Charge on big drop: Q = nq

Potential of big drop: $V = \frac{Q}{C} = \frac{nq}{n^{1/3}}V = n^{2/3}v$

Capacitance

$(c) = \frac{\text{Charge (Q)}}{\text{Potential (V)}}$

Capacitance of a parallel plate capacitor

$C = \frac{K A \varepsilon_0}{d}$ K = dielectric constant

- If a dielectric slab is partially filled between the plates.

$\Rightarrow \quad C' = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$

- If a number of dielectric slabs are inserted between the plate as shown.

$C' = \frac{\varepsilon_0 A}{d - (t_1 + t_2 + t_3 + \ldots..) + \left(\frac{t_1}{K_1} + \frac{t_2}{K_2} + \frac{t_3}{K_3} + \ldots..\right)}$

- When a metallic slab is inserted between the plates

$C' = \frac{\varepsilon_0 A}{(d - 1)}$

Energy stored in a capacitor

$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV$

Energy loss when two isolated charged conductors are connected to each other

$= \frac{1}{2}\frac{C_1 C_2 (V_1 - V_2)^2}{C_1 + C_2}$

Spherical capacitor

Spherical capacitor: It is of two concentric conducting spheres of radii a and b $(a < b)$. Inner sphere is given charge $+Q$, while outer sphere is earthed

Capacitance, $C = 4\pi\varepsilon_0 b\frac{ab}{b-a}$.

In the presence of dielectric medium (dielectric constant K) between the spheres

$C' = 4\pi\varepsilon_0 K\frac{ab}{b-a}$

If outer sphere is given a charge $+Q$ while inner sphere is earthed

Capacitance $C' = 4\pi\varepsilon_0 \cdot \frac{b^2}{b-a}$

This arrangement is not a capacitor. But it's capacitance is equivalent to the sum of capacitance of spherical capacitor and spherical conductor

i.e., $4\pi\varepsilon_0 \cdot \frac{b^2}{b-a}$

$= 4\pi\varepsilon_0 \cdot \frac{ab}{b-a} + 4\pi\varepsilon_0 b$

Cylindrical capacitor: It consists of two co-axial cylinders of radii a and b $(a < b)$, inner cylinder is given charge $+Q$ while outer cylinder is earthed. Common length of the cylinders is l

Capacitance, $C = \frac{2\pi\varepsilon_0 l}{\log_0\left(\frac{b}{a}\right)}$

Combination of capacitors

Series grouping of capacitors

Equivalent capacitance

$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}$

In series combination potential difference and energy distributes in the reverse ratio of capacitance

i.e., $V \propto \frac{1}{C}$ and $U \propto \frac{1}{C}$.

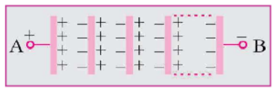
If n identical capacitors each having capacitances C are connected

$C_{eq} = \frac{C}{n}$

If n identical plates are arranged as shown they constitute $(n-1)$ capacitors in series. If each capacitors has capacitance

$\frac{\varepsilon_0 A}{d}$ then

$C_{eq} = \frac{\varepsilon_0 A}{(n-1)d}$



In this situtation except two extreme plates each plate is common to adjacent capacitors.

Here, effective capacitance C_{eq} is even less than the least of the individual capacitances.

Parallel grouping of capacitors

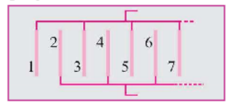
Equivalent capacitance

$C_p = C_1 + C_2 + \ldots C_n$

In parallel combination change and erergy distributes in the ratio of capacitance i.e., $Q \propto C$ and $U \propto C$. If n identical capacitors are connected in parallel, then Equivalent capacitance $C_{eq} = nC$ and Change on each capacitor

$Q' = \frac{Q}{n}$

If n identical plates are arranged such that even numbered of plates are connected together and odd numbered plates are connected together, then $(n-1)$ capacitors will be parallel grouping.



Equivalent capacitance $C' = (n-1)C$ where C = capacitance

of a capacitor $= \frac{\varepsilon_0 A}{d}$

If C_p is the effective capacity when n identical capacitors are connected in parallel and C_s is their effective capacity when connected in series, then

$\frac{C_p}{C_s} = n^2$.

Electric Charges & Fields

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ELECTRIC CHARGES & FIELDS

Charge-Due to which matter produces and experiences electric and magnetic effects.

Distribution of charge

Surface charge density, $\sigma = \frac{\text{charge}}{\text{area}}$

Linear charge density, $\lambda = \frac{\text{charge}}{\text{length}}$

Volume charge density, $\rho = \frac{\text{charge}}{\text{volume}}$

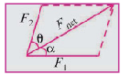
Two similar charged bodies always repel but a charged body may attract a neutral or an oppositely charged body.

Discrete distribution of change
A system consisting of ultimate individual charges.

Charge density for irregular surface is maximum where redius of curvature is minimum and vice-versa
i.e., $\sigma \propto \frac{1}{R}$

Principle of Superposition
If $q_1, q2, ... q_n$ charges applying force on a charge Q, then
$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 ..... + \vec{F}_n$
$= K \sum_{i=1}^{n} \frac{qq_i}{r_i^2} \hat{r}_i$

Magnitude of resultant of two electric forces F_1 and F_2 $F_{\text{net}} =$
$\sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos\theta}$
$\tan\alpha = \frac{F_2 \text{Sin}\,\theta}{F_1 + F_1 \cos\theta}$



Coulomb's Law
$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$
$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9\ Nm^2C^{-2}$
ϵ_0= absolute permittivity of air or free space
$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{\text{Nm}^2}$

When a dielectric medium (K) completely filled in between charges, then force between them
$F_{\text{medium}} = \frac{1}{4\pi \in_0 K} \frac{q_1 q_2}{r^2}$

When a dielectric medium (K) thickness 't' is partially filled between the charges, then force between them
$F' = \frac{1}{4 \times \varepsilon_0} \frac{q_1 q_2}{(r - t + t\sqrt{k})^2}$

Equilibrium of Charged Soap Bubble
For a charged soap bubble of radius R and surface tension T and charge density σ. The pressure due to surface tension $4\frac{T}{R}$ and atmospheric pressure P_{out} acts radially inwards and the electrical pressure (P_{el}) acts radially outwards.
The total pressure inside the soap bubble
$P_{in} = P_{out} + \frac{4T}{R} - \frac{\sigma^2}{2\varepsilon_0}$

Types of charge

Positive charge
Deficiency of electrons

Negative charge
Excess of electrons Charge of one electron $= 1.6 \times 10^{-19}$ C

Neutral No of electrons-no of protons Atoms are electrically neutral

Methods of charging

By friction or rubbing In this method, both +(ve) and –(ve) charges in equal amounts appear simultaneously due to transfer of electrons from one body to the other.

By induction Induced charge ≤ inducing charge (Q) i.e.,
$Q^1 = -Q\left(1 - \frac{1}{K}\right)$
K = dielectric constant or relative permittivity ($\in_r$)

By conduction
Two conductors one is charged and another is uncharged when bring in contact with each other, the conductors will be charged with the same sign.

Basic properties of electric charge

Quantization
$n = 1, 2, 3$ .....
$Q = +\text{ne}$

Conservation
Neither created nor destroyed

Transferable can be transferred from one body to another.

Associated with mass
Mass of electron
Me $= 9.1 \times 10^{-31}$ kg

Electric field

Electric field Space surrounding a charge in which its electrostatic force can be experienced by any test charge
$\vec{E} = \frac{\vec{F}}{q} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$

Due to descrete distribution of charge
$\vec{\text{E}} = \sum_{i=1}^{n} \vec{E}_i$

Dipole Two equal and opposite charges separated by a small distance. Dipole moment,
$\vec{P} = (q \times 2\vec{l})$

Electric field due to a dipole,

At axial position
$\text{E} = \frac{1}{4\pi\epsilon_0} \frac{2P}{r^3} \hat{r}$

At equatorial position
$E = \frac{1}{4\pi\epsilon_0} \frac{-P}{r^3} . \hat{r}$

Torque and potential energy of a dipole

Potential energy
U = PE cos θ
U = P⃗.E⃗.

Torque
τ = PE sin θ
τ = P × E

Electric field due to
(i) Point charge, $E = \frac{KQ}{r^2}$
(ii) Line charge
(a) if point where to find $\vec{E}$ at perpendicular bisector of wire then,
$E_x = \frac{2K\lambda}{r} \sin\times$ and $E_y = 0$
(b) If wire is infinitely long
$E_x = \frac{2K\lambda}{r}$ and $E_y = 0$
(c) If point lies near one end of infinitely long wire $|E_x| = |E_y| = \frac{k\lambda}{r}$
$E_{\text{net}} = \frac{\sqrt{2}K\lambda}{r}$
(iii) Changed circular ring
$E = \frac{kQx}{(x^2 + R^2)^{3/2}},\ V = \frac{kQ}{\sqrt{x^2 + R^2}}$
At centre $x = 0$ so $E_{\text{centre}} = 0$ and
$V_{\text{centre}} = \frac{KQ}{R}$
At a point on the axis such that $x >> \text{R}$
$E = \frac{KQ}{x^2},\ V = \frac{kQ}{x}$
If $x = \pm\frac{R}{\sqrt{2}}, E_{\max} = \frac{Q}{6\sqrt{3}\pi\varepsilon_0 R^2}$
and $V_{\max} - \frac{Q}{2\sqrt{6}\pi\varepsilon_0 R}$

Superposition of electric field
Resultant of electric field, due to various charges
$\vec{E} = \vec{E}_1 + \vec{E}_2 ..... + \vec{E}_n$

Due to continuous distribution of charge
$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dp}{r^3} \vec{r}$

Electric field lines
Imaginary line along which a positive test charge will move it left free.

Properties of electric field lines
- Never intersect each other
- Never from closed loops
- Come out of positive charge and go into negative charge
- Always normal to conducting surface

Electric flux
$\phi = \text{E.A} \cos\theta$
$\phi = \vec{\text{E}}.\vec{\text{A}}$

Gauss's theorem
Total flux over a closed surface is $\frac{1}{\epsilon_0}$ time the total enclosed charge
$\phi = \int E.ds = \frac{q_{\text{enclosed}}}{\varepsilon_0}$

Applicaion of Gauss's theorem
(i) Electric Field due to infinitely long straight wire
$E = \frac{\lambda}{2\pi\varepsilon_0 r}$
(ii) Infinite plane sheet
$E = \frac{\sigma}{2\varepsilon_0}$

Charged conducting shpere (or shell of charge)
(a) Outside the sphere: If point P lies outside the sphere
$E_{out} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} = \frac{\sigma R^2}{\varepsilon_0 r^2}$
(b) At the surface of sphere:
At surface $r = R$
$E_s = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} = \frac{\sigma}{\varepsilon_0}$

Infinite thin plane sheet of charge
$E = \frac{\sigma}{2\varepsilon_0} (E \propto r^\circ)$